

Online Appendix for “Joint Inference for the Regression Discontinuity Effect and Its External Validity”

Yuta Okamoto

Contents

S1 Implementation	S1
S1.1 Sharp RD	S1
S1.2 Fuzzy RD	S3
S2 Proofs and Generalization	S4
S2.1 Proofs of Lemma 1 and Proposition 1	S4
S2.2 Proofs of Lemmas 2, 3, and Proposition 2	S9
S2.3 Fuzzy RD Design	S12
S3 Simulation	S15

S1 Implementation

S1.1 Sharp RD

We describe the implementation using our first empirical example. Our implementation is based on the `rdrobust` package (Calonico et al., 2015, 2017). Users should place `extRD.R` in their working directory (or another location on the R search path) and source it. The file `extRD.R` is included in the replication package accompanying this article.

```
# Preliminaries
> library(rdrobust)
> source("extRD.R")
```

Suppose that `data1` is the data frame used in our analysis, and let `data1$outcome` and `data1$rv` denote the outcome and running variables, respectively. The `rdrobust` function implements the standard RD analysis:

```
# Standard RD
> rd_res1 <- rdrobust(data1$outcome, data1$rv)
```

The joint confidence region can be computed using the following code:

```
# Joint confidence region
> rd_joint1 <- rdrobust.joint(data1$outcome, data1$rv, rd_res1)
```

The resulting confidence region can be visualized as follows:

```
# Visualize the joint confidence region
> region.plot(rd_joint1)
```

This produces Figure 2a in the main article. To report the projection of the joint confidence region, run the following code:

```
# Report the joint confidence region
> joint.summary(rd_joint1)
Joint summary (1-alpha = 95.0%, df=2)
crit = qchisq(0.950, 2) = 5.9915

Point estimates
tau=0.2565, tau'=-0.0076

Projection of joint confidence region (simultaneous intervals)
tau      : [0.1753, 0.3701]
tau'     : [-0.0836, 0.0590]
```

Finally, one can plot the linearly extrapolated treatment-effect function, together with the uniform confidence band, using the `rd.band` function:

```
# Extrapolation
> rd.band(joint_res = rd_joint1, delta1 = -4, delta2 = 4)
```

As a result, we obtain Figure 2b.

S1.2 Fuzzy RD

In the previous subsection, we investigated the intent-to-treat effects and ignored the non-compliance issue: Those with aid offers do not necessarily accept the offer. Hence, we here study the fuzzy RD effect and its external validity.

Let `data1$fuzzy` denote the treatment status. The external validity and extrapolation analysis can be performed as follows:

```
# Fuzzy RD
> rd_res1f <- rdrobust(data1$outcome, data1$rv, fuzzy = data1$fuzzy)

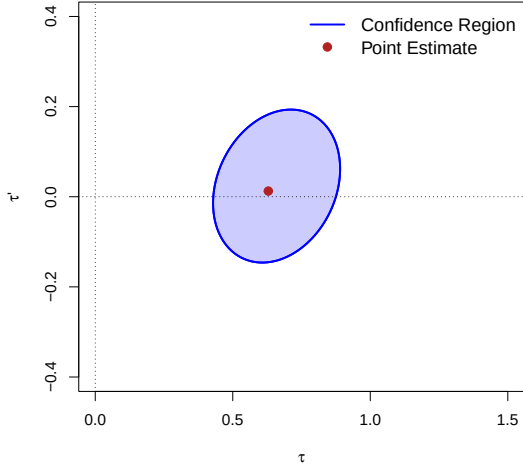
# Joint confidence region
> rd_joint1f <- rdrobust.joint(data1$outcome, data1$rv, rd_res1f,
                              fuzzy = data1$fuzzy)
> region.plot(rd_joint1f) # -> Fig S1(a)

> joint.summary(rd_joint1f)
Joint summary (1-alpha = 95.0%, df=2)
crit = qchisq(0.950, 2) = 5.9915

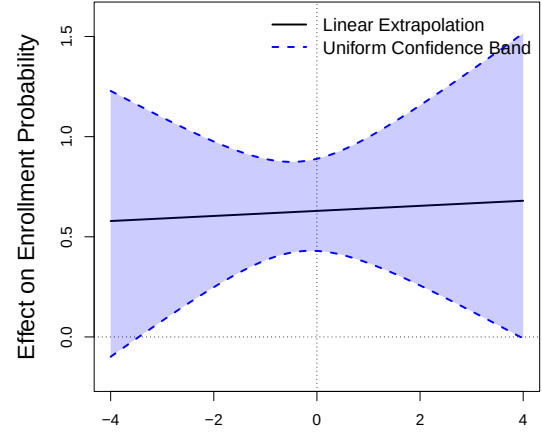
Point estimates
tau=0.6293, tau'= 0.0126

Projection of joint confidence region (simultaneous intervals)
tau      : [0.4284, 0.8903]
tau'     : [-0.1441, 0.1914]

# Extrapolation
> rd.band(joint_res=rd_joint1f, delta1=-4, delta2=4) # -> Fig S1(b)
```



(a) 95% Robust Confidence Region



(b) Linear Extrapolation and Confidence Band

Figure S1: External Validity and Extrapolation of Treatment Effects

S2 Proofs and Generalization

S2.1 Proofs of Lemma 1 and Proposition 1

This subsection provides proofs of Lemma 1 and Proposition 1 in a more general setting, where we employ $p(\geq 1)$ -th order local polynomial regression for the main estimation and use local polynomial regression of order $q(\geq p + 1)$ for the bias estimation. As the matrix notation is more convenient, we begin by introducing some additional notations. Our p -th order local polynomial estimators are given by $\hat{\mu}_+ = e_0^\top \hat{\beta}_{+,p}$ and $\hat{\mu}'_+ = e_1^\top \hat{\beta}_{+,p}$, where $e_0 := (1, 0, \dots, 0)^\top \in \mathbb{R}^{p+1}$, $e_1 := (0, 1, 0, \dots, 0)^\top \in \mathbb{R}^{p+1}$,

$$\hat{\beta}_{+,p} := \arg \min_{\beta \in \mathbb{R}^{p+1}} \sum_{i=1}^n \mathbf{1}\{X_i \geq 0\} \left(Y_i - r_p(X_i)^\top \beta \right)^2 \frac{1}{h} K\left(\frac{X_i}{h}\right),$$

and $r_p(u) := (1, u, \dots, u^p)^\top$. $\hat{\mu}_-$ and $\hat{\mu}'_-$ is defined similarly. Note that $\hat{\mu}_+$ and $\hat{\mu}'_+$ have the following closed form expression:

$$\hat{\mu}_+ = \frac{1}{n} e_0^\top \Gamma_+^{-1} R^\top W_+ Y, \quad \hat{\mu}'_+ = \frac{1}{n} \frac{1}{h} e_1^\top \Gamma_+^{-1} R^\top W_+ Y,$$

where $Y := (Y_1, \dots, Y_n)^\top$, $W_+ := \text{diag}(\mathbf{1}\{X_1 \geq 0\}K(X_1/h)/h, \dots, \mathbf{1}\{X_n \geq 0\}K(X_n/h)/h)$, $R := (r_p(X_1/h), \dots, r_p(X_n/h))^\top$, and $\Gamma_+ := R^\top W_+ R/n$. The counterparts replacing $+$ with $-$ are defined similarly. We will focus on the $+$ case as long as the $-$ case can be treated analogously below.

We assume $n \min\{h, b\} \rightarrow \infty$, $n \min\{h^{2(p+1)+1}, b^{2(p+1)+1}\} \max\{h^2, b^{2(q-p)}\} \rightarrow 0$, and $\max\{h, b\} < \kappa_0$ throughout. We also assume Assumptions 1-2 hold with $S \geq q+1 \geq p+2$.

The usual Taylor expansion argument suggests that

$$\begin{aligned} \mathbb{E} [\hat{\mu}_+ | X_1, \dots, X_n] - \mu_+ &\approx h^{p+1} \frac{\mu_+^{(p+1)}}{(p+1)!} e_0^\top \Gamma_+^{-1} \vartheta_+ =: h^{p+1} \frac{\mu_+^{(p+1)}}{(p+1)!} \mathcal{B}_+ \\ \mathbb{E} [\hat{\mu}'_+ | X_1, \dots, X_n] - \mu'_+ &\approx h^p \frac{\mu_+^{(p+1)}}{(p+1)!} e_1^\top \Gamma_+^{-1} \vartheta_+ =: h^p \frac{\mu_+^{(p+1)}}{(p+1)!} \mathcal{B}'_+, \end{aligned}$$

where $\vartheta_+ := R^\top W_+ S/n$ and $S := ((X_1/h)^{p+1}, \dots, (X_n/h)^{p+1})^\top$. We use the notations $\mathcal{B}_+$ and $\mathcal{B}'_+$, with a slight abuse of notation, to keep the exposition intuitive. This motivates our bias-corrected estimators:

$$\tilde{\mu}_+ := \hat{\mu}_+ - h^{p+1} \frac{\hat{\mu}_+^{(p+1)}}{(p+1)!} \mathcal{B}_+, \quad \text{and} \quad \tilde{\mu}'_+ := \hat{\mu}'_+ - h^p \frac{\hat{\mu}_+^{(p+1)}}{(p+1)!} \mathcal{B}'_+,$$

where $\hat{\mu}_+^{(p+1)}$ are constructed using the q -th order local polynomial regression:

$$\hat{\mu}_+^{(p+1)} := (p+1)! \frac{1}{b^{p+1}} e_{p+1}^\top \check{\Gamma}_+^{-1} \check{R}^\top \check{W}_+ Y/n,$$

where $e_{p+1} \in \mathbb{R}^{q+1}$ is the $(p+2)$ -th standard basis vector, and the checked quantities

(e.g., $\check{\Gamma}_+$) are defined by replacing p with q and h with b . Then, Lemma S.A.4 of [Calonico et al. \(2014\)](#) tells us that $\mathbb{V}[\tilde{\mu}_+|x_i, \dots, X_n] = V_+(\Sigma_Y)$ and $\mathbb{V}[h\tilde{\mu}'_+|x_i, \dots, X_n] = V'_+(\Sigma_Y)$ (with a slight abuse of notation), where

$$\begin{aligned}
V_+(\Sigma) &:= \frac{1}{n} e_0^\top \Gamma_+^{-1} (R^\top W_+ \Sigma W_+ R / n) \Gamma_+^{-1} e_0 \\
&\quad + h^{2(p+1)} \frac{1}{nb^{2(p+1)}} (p+1)!^2 e_{p+1}^\top \check{\Gamma}_+^{-1} (\check{R}^\top \check{W}_+ \Sigma \check{W}_+ \check{R} / n) \check{\Gamma}_+^{-1} e_{p+1} \frac{\mathcal{B}_+^2}{(p+1)!^2} \\
&\quad - 2h^{p+1} \frac{1}{nb^{p+1}} (p+1)! e_0^\top \Gamma_+^{-1} (R^\top W_+ \Sigma \check{W}_+ \check{R} / n) \check{\Gamma}_+^{-1} e_{p+1} \frac{\mathcal{B}_+}{(p+1)!} \\
&= \frac{1}{n} e_0^\top \Gamma_+^{-1} (R^\top W_+ \Sigma W_+ R / n) \Gamma_+^{-1} e_0 \\
&\quad + \frac{h^{2(p+1)}}{nb^{2(p+1)}} e_{p+1}^\top \check{\Gamma}_+^{-1} (\check{R}^\top \check{W}_+ \Sigma \check{W}_+ \check{R} / n) \check{\Gamma}_+^{-1} e_{p+1} \mathcal{B}_+^2 \\
&\quad - 2 \frac{h^{p+1}}{nb^{p+1}} e_0^\top \Gamma_+^{-1} (R^\top W_+ \Sigma \check{W}_+ \check{R} / n) \check{\Gamma}_+^{-1} e_{p+1} \mathcal{B}_+, \\
V'_+(\Sigma) &:= \frac{1}{n} e_1^\top \Gamma_+^{-1} (R^\top W_+ \Sigma W_+ R / n) \Gamma_+^{-1} e_1 \\
&\quad + \frac{h^{2(p+1)}}{nb^{2(p+1)}} e_{p+1}^\top \check{\Gamma}_+^{-1} (\check{R}^\top \check{W}_+ \Sigma \check{W}_+ \check{R} / n) \check{\Gamma}_+^{-1} e_{p+1} (\mathcal{B}'_+)^2 \\
&\quad - 2 \frac{h^{p+1}}{nb^{p+1}} e_1^\top \Gamma_+^{-1} (R^\top W_+ \Sigma \check{W}_+ \check{R} / n) \check{\Gamma}_+^{-1} e_{p+1} \mathcal{B}'_+,
\end{aligned}$$

and $\Sigma_Y := \text{diag}(\mathbb{V}[Y_1|X_1], \dots, \mathbb{V}[Y_n|X_n])$. Note that the only unknown quantity is Σ_Y .

We can derive a similar representation for the covariance. Note that

$$\begin{aligned}
\text{Cov}[\tilde{\mu}_+, h\tilde{\mu}'_+|X_1, \dots, X_n] &= \text{Cov}[\hat{\mu}_+ - h^{p+1}\hat{B}_+, h(\hat{\mu}'_+ - h^p\hat{B}'_+)|X_1, \dots, X_n] \\
&= h\text{Cov}[\hat{\mu}_+, \hat{\mu}'_+|X_1, \dots, X_n] - h^{p+1}\text{Cov}[\hat{\mu}_+, \hat{B}'_+|X_1, \dots, X_n] \\
&\quad - h^{p+2}\text{Cov}[\hat{B}_+, \hat{\mu}'_+|X_1, \dots, X_n] + h^{2(p+1)}\text{Cov}[\hat{B}_+, \hat{B}'_+|X_1, \dots, X_n],
\end{aligned}$$

where $\hat{B}_+ := \hat{\mu}_+^{(p+1)} / (p+1)! \times \mathcal{B}_+$ and $\hat{B}'_+ := \hat{\mu}'_+^{(p+1)} / (p+1)! \times \mathcal{B}'_+$. Then we can show

that $\text{Cov} [\tilde{\mu}_+, h\tilde{\mu}'_+ | X_1, \dots, X_n] = C_+(\Sigma_Y)$, where

$$\begin{aligned} C_+(\Sigma) &:= \frac{1}{n} e_0^\top \Gamma_+^{-1} (R^\top W_+ \Sigma W_+ R / n) \Gamma_+^{-1} e_1 \\ &\quad - \frac{h^{p+1}}{nb^{p+1}} e_0^\top \Gamma_+^{-1} (R^\top W_+ \Sigma \check{W}_+ \check{R} / n) \check{\Gamma}_+^{-1} e_{p+1} \mathcal{B}'_+ \\ &\quad - \frac{h^{p+1}}{nb^{p+1}} e_1^\top \Gamma_+^{-1} (R^\top W_+ \Sigma \check{W}_+ \check{R} / n) \check{\Gamma}_+^{-1} e_{p+1} \mathcal{B}_+ \\ &\quad + \frac{h^{2(p+1)}}{nb^{2(p+1)}} e_{p+1}^\top \check{\Gamma}_+^{-1} (\check{R}^\top \check{W}_+ \Sigma \check{W}_+ \check{R} / n) \check{\Gamma}_+^{-1} e_{p+1} \mathcal{B}_+ \mathcal{B}'_+. \end{aligned}$$

Then, by the Cramer-Wold device, it suffices to show that

$$\frac{a_0 (\tilde{\mu}_+ - \mu_+) + a_1 h (\tilde{\mu}'_+ - \mu'_+)}{\sqrt{a_0^2 V_+ + 2a_0 a_1 C_+ + a_1^2 V'_+}} \rightarrow_d \mathcal{N}(0, 1)$$

for any $(a_0, a_1)^\top \in \mathbb{R}^2 \setminus \{(0, 0)^\top\}$. Note that the left-hand side is well defined once $a_0^2 V_+ + 2a_0 a_1 C_+ + a_1^2 V'_+ > 0$, which holds with probability approaching one, as shown below. First, the same manipulation as Lemma S.A.4 of [Calonico et al. \(2014, p. 16 of Supplement\)](#) shows that

$$\begin{aligned} &\frac{a_0 (\tilde{\mu}_+ - \mu_+) + a_1 h (\tilde{\mu}'_+ - \mu'_+)}{\sqrt{a_0^2 V_+ + 2a_0 a_1 C_+ + a_1^2 V'_+}} \\ &= \underbrace{\frac{a_0 (\tilde{\mu}_+ - \mathbb{E} [\tilde{\mu}_+ | X_1, \dots, X_n]) + a_1 h (\tilde{\mu}'_+ - \mathbb{E} [\tilde{\mu}'_+ | X_1, \dots, X_n])}{\sqrt{a_0^2 V_+ + 2a_0 a_1 C_+ + a_1^2 V'_+}}}_{=:\tilde{\xi}} + o_p(1). \end{aligned}$$

Second, we will show below that $\tilde{\xi} = \tilde{\xi}_* + o_p(1)$, where $\tilde{\xi}_* := \sum_{i=1}^n \omega_{n,i} \varepsilon_i$, $\omega_{n,i} := (a_0 \omega_{0,n,i} + a_1 h \omega_{1,n,i}) / \sqrt{\omega_{2,n}}$, $\varepsilon_i = Y_i - \mu(X_i)$, $\mu(X) := \mathbb{E} [Y | X]$,

$$\begin{aligned} \omega_{0,n,i} &:= e_0^\top \Gamma_{+,*}^{-1} \frac{\mathbf{1}_i}{nh} K\left(\frac{X_i}{h}\right) r_p\left(\frac{X_i}{h}\right) - \frac{h^{p+1}}{b^{p+1}} e_0^\top \Gamma_{+,*}^{-1} \boldsymbol{\vartheta}_{+,*} e_{p+1}^\top \check{\Gamma}_{+,*}^{-1} \frac{\mathbf{1}_i}{nb} K\left(\frac{X_i}{b}\right) r_q\left(\frac{X_i}{b}\right), \\ \omega_{1,n,i} &:= e_1^\top \Gamma_{+,*}^{-1} \frac{\mathbf{1}_i}{nh^2} K\left(\frac{X_i}{h}\right) r_p\left(\frac{X_i}{h}\right) - \frac{h^p}{b^{p+1}} e_1^\top \Gamma_{+,*}^{-1} \boldsymbol{\vartheta}_{+,*} e_{p+1}^\top \check{\Gamma}_{+,*}^{-1} \frac{\mathbf{1}_i}{nb} K\left(\frac{X_i}{b}\right) r_q\left(\frac{X_i}{b}\right), \end{aligned}$$

$\mathbf{1}_i := \mathbf{1}\{X_i \geq 0\}$, $\Gamma_{+,*} := \int_0^1 K(u)r_p(u)r_p(u)^\top f(uh) du$, $\check{\Gamma}_{+,*} := \int_0^1 K(u)r_q(u)r_q(u)^\top f(ub) du$, and $\vartheta_{+,*} := \int_0^1 K(u)r_p(u)u^{p+1}f(uh) du$; and

$$\omega_{2,n} = \sum_{i=1}^n \mathbb{E} \left[(a_0\omega_{0,n,i} + a_1h\omega_{1,n,i})^2 \varepsilon_i^2 \right].$$

To see this, first observe that Lemma S.A.4 of [Calonico et al. \(2014, p. 17 of Supplement\)](#) implies that

$$\begin{aligned} & a_0 (\tilde{\mu}_+ - \mathbb{E} [\tilde{\mu}_+ | X_1, \dots, X_n]) + a_1 h (\tilde{\mu}'_+ - \mathbb{E} [\tilde{\mu}'_+ | X_1, \dots, X_n]) \\ &= \sum_{i=1}^n (a_0\omega_{0,n,i} + a_1h\omega_{1,n,i})\varepsilon_i + o_p \left((a_0^2V_+ + 2a_0a_1C_+ + a_1^2V'_+)^{1/2} \right). \end{aligned}$$

Further, we have that

$$\begin{aligned} a_0^2V_+ + 2a_0a_1C_+ + a_1^2V'_+ &= \underbrace{\sum_{i=1}^n (a_0\omega_{0,n,i} + a_1h\omega_{1,n,i})^2 \mathbb{E} [\varepsilon_i^2 | X_i]}_{=: S_n} (1 + o_p(1)) \\ &= \sum_{i=1}^n \mathbb{E} \left[(a_0\omega_{0,n,i} + a_1h\omega_{1,n,i})^2 \varepsilon_i^2 \right] (1 + o_p(1)), \end{aligned} \quad (\text{S.1})$$

where the second equality uses $S_n = \mathbb{E} [S_n] (1 + o_p(1))$, which follows from $\mathbb{V} [S_n] / \mathbb{E} [S_n]^2 = o(1)$ and the Chebyshev inequality. Hence, we obtain that $\xi = \xi_* + o_p(1)$. Again, using Lemma S.A.4 of [Calonico et al. \(2014\)](#), we obtain that $\sum_{i=1}^n \mathbb{E} [|\omega_{0,n,i}\varepsilon_i / \sqrt{\omega_{2,n}}|^4] = o(1)$ and that $\sum_{i=1}^n \mathbb{E} [|h\omega_{1,n,i}\varepsilon_i / \sqrt{\omega_{2,n}}|^4] = o(1)$. Hence, using the elementary inequality $|x + y|^4 \leq 8(|x|^4 + |y|^4)$, we obtain that $\sum_{i=1}^n \mathbb{E} [|\omega_{n,i}\varepsilon_i|^4] = o(1)$. The Liapunov central limit theorem shows $\xi_* \rightarrow_d \mathcal{N}(0, 1)$. The same argument applies to the estimators on the left of the cutoff. Because the sample is randomly drawn and the right- and left-side estimators are constructed using the mutually exclusive samples, we obtain the desired result with $V_{\text{SRD}}(\Sigma_Y) := V_+(\Sigma_Y) + V_-(\Sigma_Y)$, $V'_{\text{SRD}}(\Sigma_Y) := V'_+(\Sigma_Y) + V'_-(\Sigma_Y)$, and $C_{\text{SRD}} := C_{\text{SRD}}(\Sigma_Y) = C_+(\Sigma_Y) + C_-(\Sigma_Y)$.

Finally, we verify that Ω is asymptotically invertible. Recalling (S.1), we have that $a_0^2 V_+ + 2a_0 a_1 C_+ + a_1^2 V'_+ = \sum_{i=1}^n \mathbb{E} [(a_0 \omega_{0,n,i} + a_1 h \omega_{1,n,i})^2 \sigma_+^2(X_i)] (1 + o_p(1))$. By Assumptions 1-(i) and 1-(iii), it suffices to see that $a_0 \omega_{0,n,i} + a_1 h \omega_{1,n,i}$ is not identically zero. To see this, note that

$$a_0 \omega_{0,n,i} + a_1 h \omega_{1,n,i} = \alpha_0^\top \frac{\mathbf{1}_i}{nh} K\left(\frac{X_i}{h}\right) r_p\left(\frac{X_i}{h}\right) - \alpha_1^\top \frac{h^{p+1}}{b^{p+1}} \frac{\mathbf{1}_i}{nb} K\left(\frac{X_i}{b}\right) r_q\left(\frac{X_i}{b}\right)$$

where $\alpha_0^\top := (a_0 e_0^\top + a_1 e_1^\top) \Gamma_{+,*}^{-1}$ and $\alpha_1^\top := \alpha_0^\top \vartheta_{+,*} e_{p+1}^\top \check{\Gamma}_{+,*}^{-1}$. We first consider the case where $h = b$. Suppose, toward a contradiction, that the left-hand side is identically zero. Then, the $(p+2)$ -th element of α_1 , i.e., $\alpha_1^\top e_{p+1}$, has to be zero. However, as the matrix $\check{\Gamma}_{+,*}$ is positive definite, $e_{p+1}^\top \check{\Gamma}_{+,*}^{-1} e_{p+1} > 0$ holds, which in turn implies that $\alpha_0^\top \vartheta_{+,*} = 0$. Hence, $\alpha_1 = (0, \dots, 0)^\top$, thereby $\alpha_0 = (0, \dots, 0)^\top$ should hold. Since $\Gamma_{+,*}^{-1}$ is invertible, this holds only when $(a_0, a_1) = (0, 0)$, which contradicts our assumption. When $h/b \rightarrow \rho$ with $\rho < 1$, noting that $K(x/h) = 0$ on $x \in (h, b]$ for large n enough, we can show that $\alpha_1 = (0, \dots, 0)^\top$, thereby $\alpha_0 = (0, \dots, 0)^\top$. The case where $\rho > 1$ is similar. Hence, $a_0^2 V_+ + 2a_0 a_1 C_+ + a_1^2 V'_+ > 0$ with probability approaching one for any $(a_0, a_1)^\top \in \mathbb{R}^2 \setminus \{(0, 0)^\top\}$. The same argument applies to the estimators on the left of the cutoff, and hence we obtain the desired result.

Proof of Lemma 1. This lemma is a special case of the argument above. □

Proof of Proposition 1. This proposition is a corollary to the previous lemma. □

S2.2 Proofs of Lemmas 2, 3, and Proposition 2

Proof of Lemma 2. Under the differentiability condition, τ_{SRD} and τ'_{SRD} are both identified (Dong and Lewbel, 2015, Theorem 1). Then, a linear function over an interval including $x = 0$ is identified. Hence, Assumption 3 implies the identification. □

Proof of Proposition 2. Lemma 1 have shown that $\tilde{\Delta}(\tau_{\text{SRD}}, \tau'_{\text{SRD}}) \rightarrow_d \mathcal{N}((0, 0)^\top, \Omega_h)$. Then,

we have that

$$\frac{(1, x) \tilde{\Delta}(\tau_{\text{SRD}}, \tau'_{\text{SRD}})}{\sqrt{(1, x) \Omega_h(1, x)^\top}} \rightarrow_d v(x)^\top Z, \text{ where } Z \sim \mathcal{N}\left((0, 0)^\top, \text{diag}(1, 1)\right)$$

and $v(x) := \Omega_h^{1/2}(1, x)^\top / \|\Omega_h^{1/2}(1, x)^\top\|$. We are interested in the limit behavior of the supremum of the left-hand side, $\sup_{x \in [-\delta_1, \delta_2]} |(1, x) \tilde{\Delta}(\tau_{\text{SRD}}, \tau'_{\text{SRD}}) / \sqrt{(1, x) \Omega_h(1, x)^\top}|$. By the continuous mapping, its limit is $S := \sup_{x \in [-\delta_1, \delta_2]} |v(x)^\top Z|$. By transforming to polar coordinates, we can decompose the normal variable Z as $Z = R(\cos \phi, \sin \phi)^\top$, where R follows the standard Rayleigh distribution, $\phi \sim \text{Uniform}[0, 2\pi)$, and $R \perp \phi$. Hence, we have $\sup_{x \in [-\delta_1, \delta_2]} |v(x)^\top Z| = R \cdot \sup_{x \in [-\delta_1, \delta_2]} |v(x)^\top (\cos \phi, \sin \phi)^\top|$.

Now, recalling that the definition of $v(x)$, it is on the unit circle. To study the direction of $v(x)$, we define the non-normalized version $u(x) := \Omega_h^{1/2}(1, x)^\top$, and identify $\mathbb{R}^2$ with the complex plane so that we examine the argument of the complex number $u_1(x) + iu_2(x)$, where $u(x) = (u_1(x), u_2(x))^\top$. By construction, there exist real constants a_1, a_2, b_1, b_2 such that $u(x) = (a_1 + b_1x, a_2 + b_2x)^\top$. Then we compute:

$$\frac{\partial}{\partial x} \arg(u_1(x) + iu_2(x)) = \frac{a_1b_2 - a_2b_1}{(a_1 + b_1x)^2 + (a_2 + b_2x)^2} > 0,$$

since $a_1b_2 - a_2b_1 = \det(\Omega_h^{1/2}) > 0$ by the regularity of Ω_h . Hence, the argument of $u(x)$ is strictly increasing in x , and so is $v(x)$. Therefore, as x ranges over $[-\delta_1, \delta_2]$, $v(x)$ traces out an arc of the unit circle. Let $\theta(x)$ denote this argument, and it is easy to see that there exists some closed interval $[\theta_l, \theta_u]$ such that $\theta(x) \in [\theta_l, \theta_u]$, $\theta_u - \theta_l \leq \pi$, and the boundary

points are attained at $x = -\delta_1$ or $x = \delta_2$. Then, we can rewrite that

$$\begin{aligned}
S &= R \cdot \sup_{x \in [-\delta_1, \delta_2]} |v(x)^\top (\cos \phi, \sin \phi)^\top| \\
&= R \cdot \sup_{x \in [-\delta_1, \delta_2]} |\cos(\theta(x) - \phi)| \\
&= R \cdot \sup_{\theta \in [\theta_l, \theta_u]} |\cos(\theta - \phi)|,
\end{aligned}$$

where the second equality uses the cosine addition formula. The distribution of the last term is equal to that of $R \cdot \sup_{\theta \in [0, \theta_u - \theta_l]} |\cos(\theta - \phi)|$, whose distribution is again equivalent to the distribution of $R \cdot \sup_{\theta \in [0, \theta_u - \theta_l]} |\cos(\theta - \varphi)|$, where $\varphi \sim \text{Uniform}[0, \pi]$. Then, we can further rewrite this as

$$R \cdot \begin{cases} 1 & \text{if } \varphi \in [0, \theta_u - \theta_l] =: I_1 \\ |\cos((\theta_u - \theta_l) - \varphi)| & \text{if } \varphi \in [\theta_u - \theta_l, \pi/2 + (\theta_u - \theta_l)/2] =: I_2 \\ |\cos(-\varphi)| & \text{if } \varphi \in [\pi/2 + (\theta_u - \theta_l)/2, \pi] =: I_3. \end{cases}$$

Figure S2 must be instructive to derive this. Again, by observing Figure S2, we finally obtain that

$$\begin{aligned}
S &= R \cdot \begin{cases} 1 & \text{with probability } (\theta_u - \theta_l)/\pi \\ \cos(U) & \text{with probability } 1 - (\theta_u - \theta_l)/\pi, \end{cases} \\
U &\sim \text{Uniform}[0, \pi/2 - (\theta_u - \theta_l)/2], \quad R \perp U.
\end{aligned}$$

Hence, the distribution function of S is given by

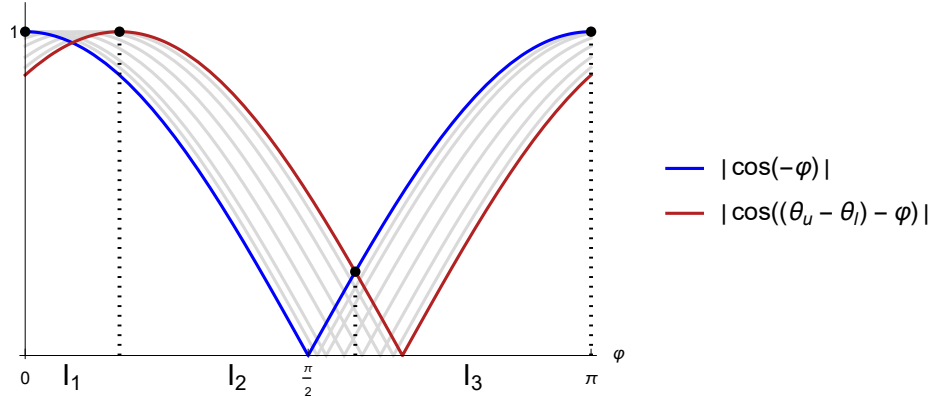


Figure S2: Illustration of I_1 , I_2 , and I_3

$$\begin{aligned}
& \mathbb{P}[S \leq s] \\
&= \frac{\theta_u - \theta_l}{\pi} \mathbb{P}[R \leq s] + \left(1 - \frac{\theta_u - \theta_l}{\pi}\right) \int_0^{\pi/2 - (\theta_u - \theta_l)/2} \mathbb{P}\left[R \leq \frac{s}{\cos(u)}\right] \frac{2}{\pi - (\theta_u - \theta_l)} du \\
&= \frac{\theta_u - \theta_l}{\pi} \left[1 - \exp\left(-\frac{s^2}{2}\right)\right] + \frac{2}{\pi} \int_0^{\pi/2 - (\theta_u - \theta_l)/2} 1 - \exp\left(-\frac{s^2}{2 \cos(u)^2}\right) du.
\end{aligned}$$

Thus, combined with the consistency of $\hat{\Omega}$, the critical value defined in the proposition provides asymptotically correct coverage. $\square$

Proof of Lemma 3. If $[-\delta_1, \delta_2] \supseteq [-\delta'_1, \delta'_2]$, corresponding ℓ satisfies $\ell \geq \ell'$. Then, it suffices to see that the critical value is nondecreasing in ℓ . Rewrite

$$P(s; \ell) = \frac{\ell}{\pi} \left[1 - \exp\left(-\frac{s^2}{2}\right)\right] + \frac{2}{\pi} \int_0^{\pi/2 - \ell/2} 1 - \exp\left(-\frac{s^2}{2 \cos(u)^2}\right) du.$$

By the Leibniz rule, we can show that $\partial P(s; \ell) / \partial \ell \leq 0$. It is also straightforward to see that $\partial P(s; \ell) / \partial s > 0$ for $s > 0$. Then, the implicit function theorem proves the statement. $\square$

S2.3 Fuzzy RD Design

Extension of our proposals to the fuzzy RD case is straightforward. Suppose a random sample $(Y_i(0), Y_i(1), T_i(0), T_i(1), X_i)$. Here, $T(0)$ and $T(1)$ denote the potential treatment

indicator, and we observe $T_i = T_i(1)\mathbf{1}\{X_i \geq 0\} + T_i(0)\mathbf{1}\{X_i < 0\}$. In fuzzy designs, our target parameters are $\tau_{\text{FRD}} := \mathbb{E}[Y_i(1) - Y_i(0)|X_i = 0, T_i(1) > T_i(0)]$ and its derivative with respect to x , τ'_{FRD} . Under Assumption A3 of [Dong and Lewbel \(2015\)](#), these are identified as

$$\tau_{\text{FRD}} = \frac{\tau_Y}{\tau_T} := \frac{\mu_{Y,+} - \mu_{Y,-}}{\mu_{T,+} - \mu_{T,-}}, \quad \tau'_{\text{FRD}} = \frac{\tau'_Y}{\tau_T} - \frac{\tau'_T}{\tau_T} \cdot \frac{\tau_Y}{\tau_T} = \frac{\tau'_Y \tau_T - \tau_Y \tau'_T}{\tau_T^2},$$

where $\mu_{A,+} := \lim_{x \downarrow 0} \mu_A(x)$, $\mu_{A,-} := \lim_{x \uparrow 0} \mu_A(x)$, and $\mu_A(x) := \mathbb{E}[A_i|X_i = x]$, where A equals either Y or T . $\tau_T \neq 0$ is assumed.

In addition to the assumptions made in Lemma 1, we assume the following:

Assumption S2.1. $h \rightarrow 0$. Furthermore, in a neighbourhood $(-\kappa_0, \kappa_0)$ around the cutoff:

- (i) $\mu_{T,+}(x) := \mathbb{E}[T_i(1)|X_i = x]$ and $\mu_{T,-}(x) := \mathbb{E}[T_i(0)|X_i = x]$ are S -times continuously differentiable.
- (ii) $\sigma_{T,+}^2(x) := \mathbb{V}[T_i(1)|X_i = x]$ and $\sigma_{T,-}^2(x) := \mathbb{V}[T_i(0)|X_i = x]$ are continuous and bounded away from zero.

Below, we suppose that the assumptions made in Section S2.1 and Assumption S2.1 are satisfied. Let $\hat{\tau}_{\text{FRD}}$ and $\hat{\tau}'_{\text{FRD}}$ be based on the p -th order local polynomial estimators using the same bandwidth h for every estimation step. The bias-corrected estimator of τ_{FRD} is given by ([Calonico et al., 2014](#), Supplement, p.24)

$$\tilde{\tau}_{\text{FRD}} = \hat{\tau}_{\text{FRD}} - h^{p+1} \left[\left\{ \frac{1}{\hat{\tau}_T} \frac{\hat{\mu}_{Y,+}^{(p+1)}}{(p+1)!} - \frac{\hat{\tau}_Y}{\hat{\tau}_T^2} \frac{\hat{\mu}_{T,+}^{(p+1)}}{(p+1)!} \right\} \mathcal{B}_+ - \left\{ \frac{1}{\hat{\tau}_T} \frac{\hat{\mu}_{Y,-}^{(p+1)}}{(p+1)!} - \frac{\hat{\tau}_Y}{\hat{\tau}_T^2} \frac{\hat{\mu}_{T,-}^{(p+1)}}{(p+1)!} \right\} \mathcal{B}_- \right].$$

Similarly, we can define the bias-corrected estimator of τ'_{FRD} by

$$\tilde{\tau}'_{\text{FRD}} = \hat{\tau}'_{\text{FRD}} - h^p \left[\left\{ \frac{1}{\hat{\tau}_T} \frac{\hat{\mu}_{Y,+}^{(p+1)}}{(p+1)!} - \frac{\hat{\tau}_Y}{\hat{\tau}_T^2} \frac{\hat{\mu}_{T,+}^{(p+1)}}{(p+1)!} \right\} \mathcal{B}'_+ - \left\{ \frac{1}{\hat{\tau}_T} \frac{\hat{\mu}_{Y,-}^{(p+1)}}{(p+1)!} - \frac{\hat{\tau}_Y}{\hat{\tau}_T^2} \frac{\hat{\mu}_{T,-}^{(p+1)}}{(p+1)!} \right\} \mathcal{B}'_- \right].$$

It is useful to rewrite (Calonico et al., 2014, Supplement, p. 24):

$$\tilde{\tau}_{\text{FRD}} - \tau_{\text{FRD}} = \underbrace{\frac{1}{\tau_T} (\tilde{\tau}_Y - \tau_Y) - \frac{\tau_Y}{\tau_T^2} (\tilde{\tau}_T - \tau_T)}_{=:\hat{\tau}_{\text{FRD}}} + R_n - R_n^{\text{bc}},$$

where R_n and R_n^{bc} are given in Calonico et al. (2014, Supplement, p. 24) and are shown to be asymptotically negligible. Similarly, one can show that

$$\tilde{\tau}'_{\text{FRD}} - \tau'_{\text{FRD}} = \underbrace{\frac{1}{\tau_T} (\tilde{\tau}'_Y - \tau'_Y) - \frac{\tau_Y}{\tau_T^2} (\tilde{\tau}'_T - \tau'_T)}_{=:\hat{\tau}'_{\text{FRD}}} + R'_n - (R'_n)^{\text{bc}},$$

where $(R'_n)^{\text{bc}}$ is analogously defined as R_n^{bc} , and R'_n is given by

$$R'_n := \frac{\hat{\tau}'_Y}{\hat{\tau}_T \tau_T} (\tau_T - \hat{\tau}_T) - \frac{\hat{\tau}'_T \hat{\tau}_Y}{\hat{\tau}_T^2 \tau_T^2} (\tau_T^2 - \hat{\tau}_T^2) - \frac{\hat{\tau}'_T}{\tau_T^2} (\hat{\tau}_Y - \tau_Y)$$

Then, $(R'_n)^{\text{bc}}$ and R'_n can be shown to be of smaller order analogously. The conditional variance of $\hat{\tau}_{\text{FRD}}$ is derived by Calonico et al. (2014, Theorem A.2), which is written by V_{FRD} :

$$V_{\text{FRD}} := \frac{1}{\tau_T^2} V_{\text{SRD}}(\Sigma_Y) - 2 \frac{\tau_Y}{\tau_T^3} V_{\text{SRD}}(\Sigma_{Y,T}) + \frac{\tau_Y^2}{\tau_T^4} V_{\text{SRD}}(\Sigma_T)$$

where $\Sigma_{Y,T} := \text{diag}(\text{Cov}[Y_1, T_1|X_1], \dots, \text{Cov}[Y_n, T_n|X_n])$ and Σ_T is defined analogously to Σ_Y . The conditional variance of $h\hat{\tau}'_{\text{FRD}}$ can be similarly obtained as provided below:

$$V'_{\text{FRD}} := \frac{1}{\tau_T^2} V'_{\text{SRD}}(\Sigma_Y) - 2 \frac{\tau_Y}{\tau_T^3} V'_{\text{SRD}}(\Sigma_{Y,T}) + \frac{\tau_Y^2}{\tau_T^4} V'_{\text{SRD}}(\Sigma_T).$$

Observing that $\hat{\tau}_{\text{FRD}}^{(\nu)}$ is a (weighted) difference of the two bias-corrected sharp RD esti-

mands, the covariance can be computed as $\text{Cov} [\tau_{\text{FRD}}^\circ, h\tau_{\text{FRD}}^{\circ'} | X_1, \dots, X_n] = \mathbf{C}_{\text{FRD}}$, where

$$\mathbf{C}_{\text{FRD}} := \frac{1}{\tau_T^2} \mathbf{C}_{\text{SRD}}(\Sigma_Y) - \frac{\tau_Y}{\tau_T^3} \mathbf{C}_{\text{SRD}}(\Sigma_{Y,T}) - \frac{\tau_Y}{\tau_T^3} \mathbf{C}_{\text{SRD}}(\Sigma_{T,Y}) + \frac{\tau_Y^2}{\tau_T^4} \mathbf{C}_{\text{SRD}}(\Sigma_T),$$

Given this result, a confidence region analogous to that in Proposition 1 can be readily constructed:

$$\Omega_{\text{FRD}}^{-1/2} \text{diag}(1, h) \tilde{\Delta}(\tau_{\text{FRD}}, \tau_{\text{FRD}}') \rightarrow_d \mathcal{N} \left((0, 0)^\top, \text{diag}(1, 1) \right), \quad \Omega_{\text{FRD}} := \begin{pmatrix} \mathbf{V}_{\text{FRD}} & \mathbf{C}_{\text{FRD}} \\ \mathbf{C}_{\text{FRD}} & \mathbf{V}_{\text{FRD}}' \end{pmatrix}.$$

Instead of Assumption 3 in the main text, we can assume the local linearity of $\mathbb{E}[Y_i(1) - Y_i(0) | X_i = x, T_i(1) > T_i(0)]$ in $x \in [-\delta_1, \delta_2]$, thereby obtaining an identification result and a confidence band analogous to those in Lemma 2 and Proposition 2.

S3 Simulation

The data-generating processes used in the simulation studies are provided in Figure S3. Additional simulation results are summarized in Tables S1-S3.

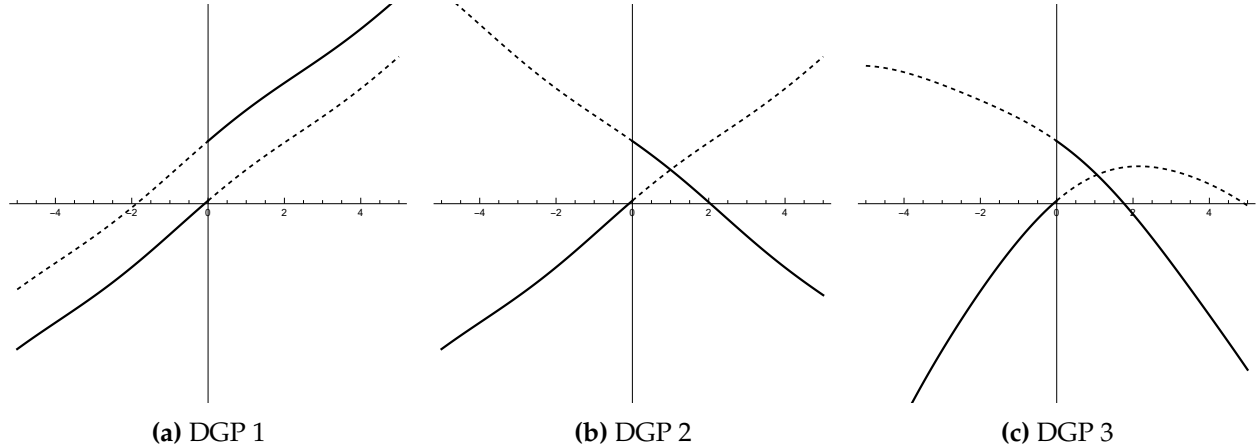


Figure S3: Data Generating Processes

	Local Linear ($p = 1$)					Local Quadratic ($p = 2$)				
	$\hat{h}_{\text{mean}}$	$\hat{h}_{\text{deriv}}$	$\hat{h}_{\text{both}}$	$\hat{h}_{\text{reg0}}$	$\hat{h}_{\text{reg1}}$	$\hat{h}_{\text{mean}}$	$\hat{h}_{\text{deriv}}$	$\hat{h}_{\text{both}}$	$\hat{h}_{\text{reg0}}$	$\hat{h}_{\text{reg1}}$
DGP 1: Coverage										
$\mathcal{R}_{0.95}$	0.930	0.928	0.930	0.930	0.929	0.929	0.930	0.929	0.928	0.928
$\mathcal{U}_{0.95}(\delta = 0.05)$	0.939	0.941	0.939	0.938	0.935	0.920	0.922	0.920	0.920	0.924
$\mathcal{U}_{0.95}(\delta = 1)$	0.931	0.927	0.927	0.927	0.929	0.927	0.929	0.925	0.927	0.926
$\mathcal{U}_{0.95}(\delta = 5)$	0.930	0.928	0.928	0.930	0.929	0.929	0.930	0.930	0.928	0.928
DGP 1: MISE										
$\delta = 0.05$	0.027	0.029	0.027	0.019	0.020	0.099	0.108	0.098	0.032	0.032
$\delta = 1$	2.045	1.840	1.556	1.292	1.513	7.115	6.112	4.903	3.409	3.746
$\delta = 5$	190.9	159.8	145.1	116.1	141.7	653.4	506.2	473.4	349.8	391.3
DGP 2: Coverage										
$\mathcal{R}_{0.95}$	0.934	0.930	0.933	0.927	0.929	0.934	0.932	0.933	0.934	0.935
$\mathcal{U}_{0.95}(\delta = 0.05)$	0.946	0.944	0.946	0.933	0.936	0.930	0.926	0.929	0.925	0.929
$\mathcal{U}_{0.95}(\delta = 1)$	0.932	0.926	0.929	0.927	0.927	0.933	0.931	0.933	0.933	0.934
$\mathcal{U}_{0.95}(\delta = 5)$	0.934	0.930	0.931	0.927	0.929	0.934	0.933	0.933	0.934	0.935
DGP 2: MISE										
$\delta = 0.05$	0.028	0.026	0.028	0.020	0.021	0.086	0.105	0.084	0.033	0.033
$\delta = 1$	2.037	1.690	1.587	1.291	1.521	6.524	6.128	4.847	3.470	3.887
$\delta = 5$	188.7	148.1	144.7	114.4	141.0	611.5	515.6	494.5	356.4	408.3
DGP 3: Coverage										
$\mathcal{R}_{0.95}$	0.933	0.933	0.933	0.929	0.930	0.912	0.923	0.910	0.915	0.915
$\mathcal{U}_{0.95}(\delta = 0.05)$	0.940	0.943	0.940	0.935	0.931	0.916	0.923	0.916	0.910	0.904
$\mathcal{U}_{0.95}(\delta = 1)$	0.931	0.931	0.929	0.927	0.928	0.911	0.923	0.920	0.914	0.915
$\mathcal{U}_{0.95}(\delta = 5)$	0.910	0.905	0.904	0.895	0.912	0.906	0.910	0.907	0.901	0.906
DGP 3: MISE										
$\delta = 0.05$	0.026	0.023	0.026	0.021	0.022	0.090	0.135	0.089	0.035	0.035
$\delta = 1$	2.042	1.526	1.403	1.403	1.619	7.639	7.086	5.422	3.882	4.340
$\delta = 5$	205.7	148.5	142.6	138.4	161.8	752.0	574.6	553.3	415.3	471.8

Table S1: Main Simulation Results ($n = 500$)

	Local Linear ($p = 1$)					Local Quadratic ($p = 2$)				
	$\hat{h}_{\text{mean}}$	$\hat{h}_{\text{deriv}}$	$\hat{h}_{\text{both}}$	$\hat{h}_{\text{reg0}}$	$\hat{h}_{\text{reg1}}$	$\hat{h}_{\text{mean}}$	$\hat{h}_{\text{deriv}}$	$\hat{h}_{\text{both}}$	$\hat{h}_{\text{reg0}}$	$\hat{h}_{\text{reg1}}$
DGP 1: MSE at $x = -\delta_1$										
$\delta = 0.05$	0.138	0.163	0.137	0.105	0.112	0.481	0.584	0.474	0.176	0.181
$\delta = 1$	1.170	1.029	0.942	0.784	0.964	4.732	3.897	3.278	2.503	2.898
$\delta = 5$	25.618	21.033	20.342	16.609	20.619	105.528	78.318	74.746	57.426	66.819
DGP 1: MSE at $x = \delta_2$										
$\delta = 0.05$	0.135	0.159	0.135	0.103	0.107	0.474	0.556	0.465	0.171	0.173
$\delta = 1$	1.128	0.941	0.887	0.730	0.873	4.576	3.336	3.066	2.391	2.744
$\delta = 5$	25.405	20.595	19.996	16.336	20.166	104.745	75.513	73.086	56.862	66.051
DGP 2: MSE at $x = -\delta_1$										
$\delta = 0.05$	0.132	0.147	0.132	0.103	0.109	0.381	0.516	0.377	0.169	0.171
$\delta = 1$	1.116	0.946	0.878	0.771	0.924	3.906	3.212	2.765	2.216	2.469
$\delta = 5$	23.901	19.925	19.159	16.533	20.175	86.109	68.590	65.702	51.052	57.636
DGP 2: MSE at $x = \delta_2$										
$\delta = 0.05$	0.128	0.146	0.127	0.101	0.107	0.367	0.519	0.363	0.167	0.171
$\delta = 1$	1.023	0.920	0.842	0.737	0.886	3.630	3.266	2.667	2.171	2.457
$\delta = 5$	23.440	19.791	19.005	16.361	19.988	84.728	68.857	65.528	50.826	57.574
DGP 3: MSE at $x = -\delta_1$										
$\delta = 0.05$	0.142	0.115	0.139	0.097	0.106	0.453	0.504	0.447	0.168	0.172
$\delta = 1$	1.232	0.891	0.816	0.745	0.924	4.179	3.577	3.031	2.394	2.707
$\delta = 5$	32.871	24.670	24.193	22.199	26.828	99.954	83.566	78.624	62.851	70.517
DGP 3: MSE at $x = \delta_2$										
$\delta = 0.05$	0.139	0.114	0.136	0.099	0.108	0.452	0.506	0.447	0.171	0.174
$\delta = 1$	1.166	0.895	0.825	0.784	0.956	4.156	3.614	2.939	2.445	2.747
$\delta = 5$	32.486	26.570	25.735	23.412	27.140	99.134	84.356	78.004	62.250	69.845

Table S2: Simulation Results: MSE ($n = 1000$)

	Local Linear ($p = 1$)					Local Quadratic ($p = 2$)				
	$\hat{h}_{\text{mean}}$	$\hat{h}_{\text{deriv}}$	$\hat{h}_{\text{both}}$	$\hat{h}_{\text{reg0}}$	$\hat{h}_{\text{reg1}}$	$\hat{h}_{\text{mean}}$	$\hat{h}_{\text{deriv}}$	$\hat{h}_{\text{both}}$	$\hat{h}_{\text{reg0}}$	$\hat{h}_{\text{reg1}}$
DGP 1: MSE at $x = -\delta_1$										
$\delta = 0.05$	0.274	0.297	0.273	0.190	0.199	0.993	1.080	0.981	0.326	0.329
$\delta = 1$	2.500	2.175	1.837	1.506	1.826	8.523	6.847	5.742	4.434	4.921
$\delta = 5$	56.601	47.346	42.868	34.188	41.877	193.129	148.859	139.602	104.086	116.461
DGP 1: MSE at $x = \delta_2$										
$\delta = 0.05$	0.277	0.297	0.276	0.195	0.204	1.012	1.098	1.001	0.331	0.335
$\delta = 1$	2.557	2.175	1.983	1.613	1.923	8.888	7.193	6.066	4.521	5.034
$\delta = 5$	56.889	47.347	43.324	34.725	42.364	194.956	150.590	140.784	104.518	117.026
DGP 2: MSE at $x = -\delta_1$										
$\delta = 0.05$	0.278	0.268	0.277	0.199	0.210	0.874	1.070	0.861	0.334	0.337
$\delta = 1$	2.450	2.015	1.954	1.552	1.889	8.215	7.343	5.985	4.588	5.220
$\delta = 5$	55.774	43.933	43.020	33.970	41.981	182.397	153.794	147.284	106.407	122.036
DGP 2: MSE at $x = \delta_2$										
$\delta = 0.05$	0.283	0.267	0.282	0.199	0.208	0.862	1.046	0.847	0.331	0.333
$\delta = 1$	2.562	2.003	1.927	1.539	1.856	7.957	6.868	5.760	4.531	5.148
$\delta = 5$	56.330	43.872	42.899	33.905	41.821	181.106	151.419	145.708	106.122	121.676
DGP 3: MSE at $x = -\delta_1$										
$\delta = 0.05$	0.267	0.230	0.264	0.210	0.225	0.908	1.360	0.893	0.355	0.357
$\delta = 1$	2.648	1.814	1.702	1.721	2.018	9.492	7.885	6.554	5.190	5.825
$\delta = 5$	64.520	44.583	43.242	42.644	50.079	226.684	172.470	166.542	127.670	144.265
DGP 3: MSE at $x = \delta_2$										
$\delta = 0.05$	0.258	0.231	0.256	0.207	0.222	0.927	1.366	0.905	0.351	0.358
$\delta = 1$	2.460	1.868	1.720	1.680	1.971	9.853	7.995	6.578	5.098	5.825
$\delta = 5$	62.855	48.600	46.502	44.561	51.130	225.970	171.903	165.509	125.132	142.466

Table S3: Simulation Results: MSE ($n = 500$)

References

- Calonico, S., Cattaneo, M. D., Farrell, M. H., and Titiunik, R. (2017). `rdrobust`: Software for Regression-discontinuity Designs. *The Stata Journal*, 17(2):372–404.
- Calonico, S., Cattaneo, M. D., and Titiunik, R. (2014). Robust Nonparametric Confidence Intervals for Regression-Discontinuity Designs. *Econometrica*, 82(6):2295–2326.
- Calonico, S., Cattaneo, M. D., and Titiunik, R. (2015). `rdrobust`: An R Package for Robust Nonparametric Inference in Regression-Discontinuity Designs. *The R Journal*, 7:38–51.
- Dong, Y. and Lewbel, A. (2015). Identifying the Effect of Changing the Policy Threshold in Regression Discontinuity Models. *The Review of Economics and Statistics*, 97(5):1081–1092.